7.2 - Inverse Transforms and Transforms of Derivatives

Theorem 7.2.1: Some Inverse Transforms

$$\text{(a) } 1 = \mathscr{L}^{-1}\left\{\frac{1}{s}\right\}$$

(b) $t^n = \mathscr{L}^{-1}\left\{\dfrac{n!}{s^{n+1}}\right\}, \; n = 1,\ 2,\ 3,\ \ldots$

(c) $e^{at} = \mathscr{L}^{-1}\left\{\dfrac{1}{s-a}\right\}$

(d) $\sin kt = \mathscr{L}^{-1}\left\{\dfrac{k}{s^2+k^2}\right\}$

(e) $\cos kt = \mathscr{L}^{-1}\left\{\dfrac{s}{s^2+k^2}\right\}$

(f) $\sinh kt = \mathscr{L}^{-1}\left\{\dfrac{k}{s^2-k^2}\right\}$

(g) $\cosh kt = \mathscr{L}^{-1}\left\{\dfrac{s}{s^2-k^2}\right\}$

Example: Find the inverse Laplace transform.

$\mathscr{L}^{-1}\left\{\dfrac{1}{s^4}\right\}$

$\mathscr{L}^{-1}\left\{\frac{10s}{s^2+16}\right\}$

$\mathscr{L}^{-1}\left\{\frac{s+1}{s^2+2}\right\}$

$\mathscr{L}^{-1}\left\{\frac{s+1}{s^2-4s}\right\}$

Example: Use the Laplace transform to solve the Initial-Value Problem.

$$2\frac{dy}{dt} + y = 0, \qquad y(0) = -3$$

Transforms of derivatives

Theorem: Transform of a Derivative

If $f,\ f',\ \ldots,\ f^{(n-1)}$ are continuous on $[0,\ \infty)$ and are of exponential order and if $f^{(n)}(t)$ is piecewise continuous on $[0,\ \infty)$, then

$$\mathscr{L}\left\{f^{(n)}(t)\right\} = s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) - \cdots - f^{(n-1)}(0),$$

where $F(s) = \mathscr{L}\{f(t)\}$.

Example: Use the Laplace transform to solve the Initial-Value Problem.

$y'' - 4y' = 6e^{3t} - 3e^{-t}, \qquad y(0) = 1, \quad y'(0) = -1$